Mersenne factorization factory

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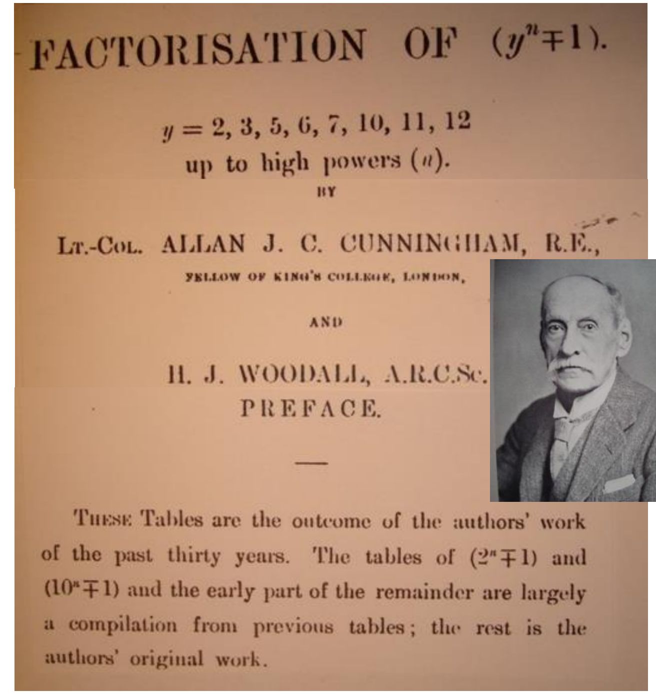
Integer factorization: why?

traditionally:

mostly for recreational purposes



FACTORISATION OF $(y^n \mp 1)$.

$y = 2, 3, 5, 6, 7, 10, 11, 12$
up to high powers (n).

BY

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AND

H. J. WOODALL, A.R.C.Sc.

PREFACE.

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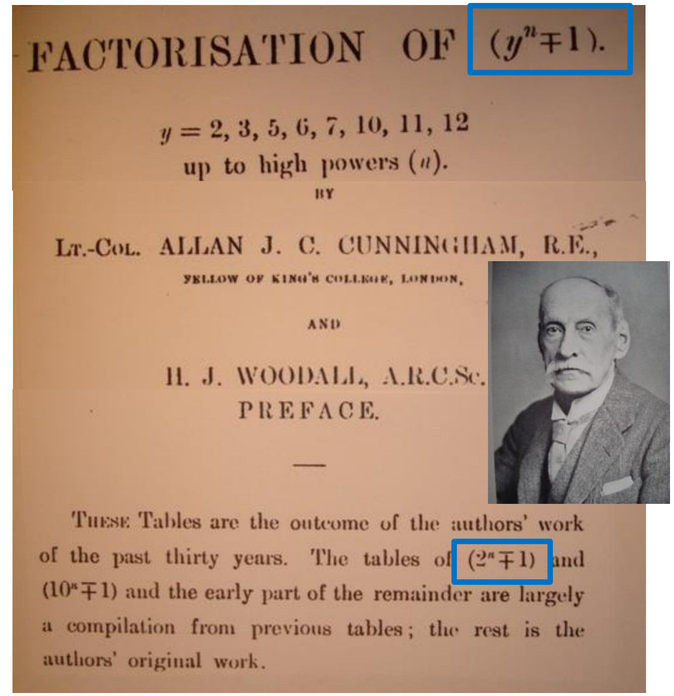

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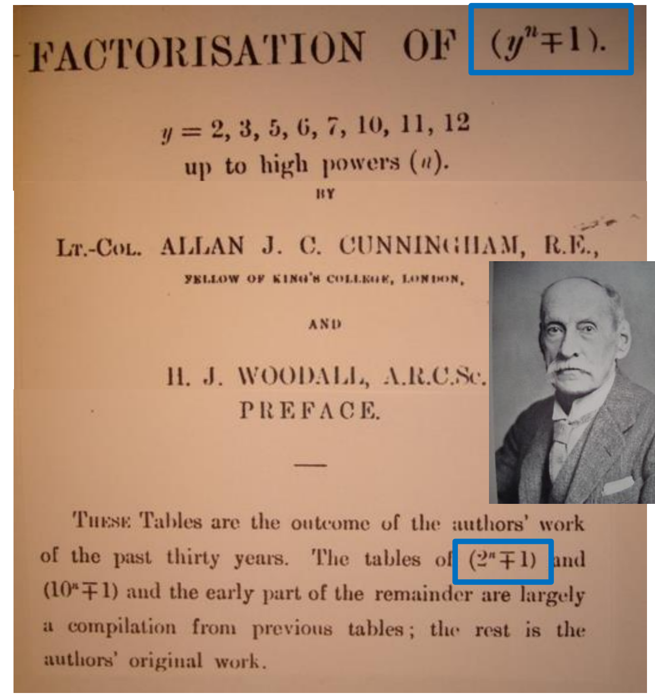
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- currently believed to be easier:
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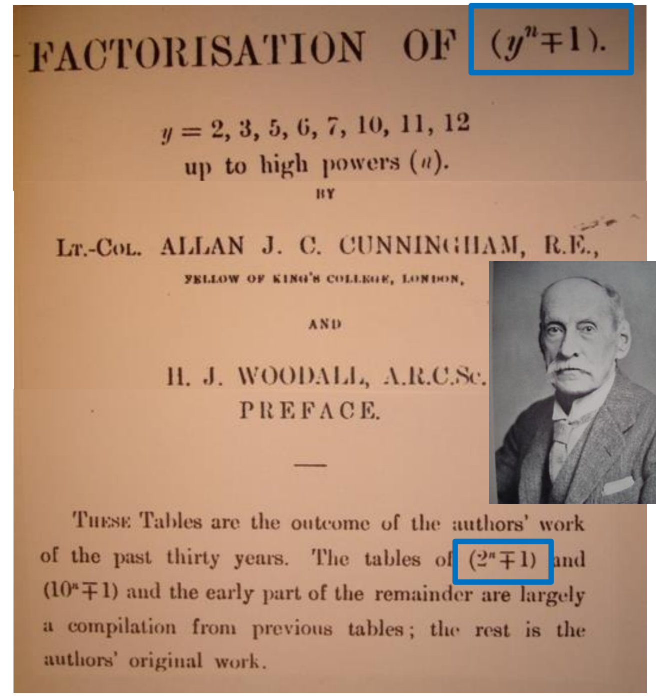
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- currently believed to be the hardest case:
  nothing better published than number field sieve (NFS)

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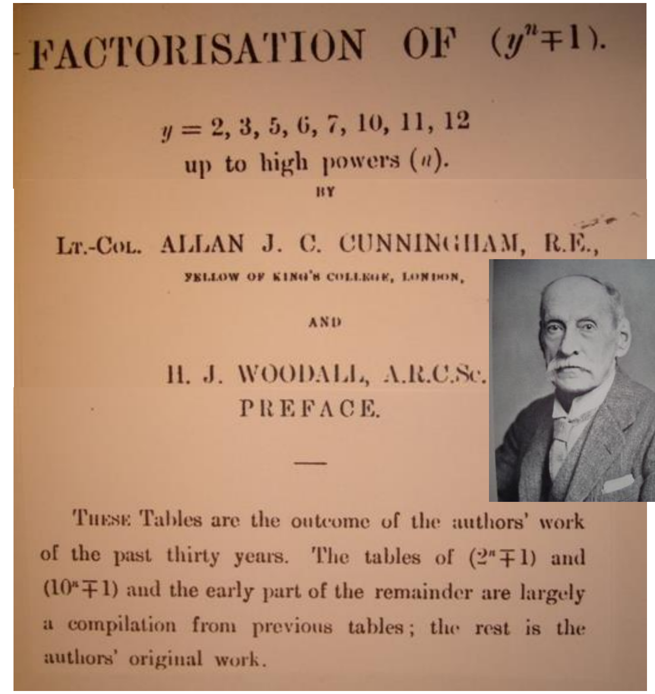
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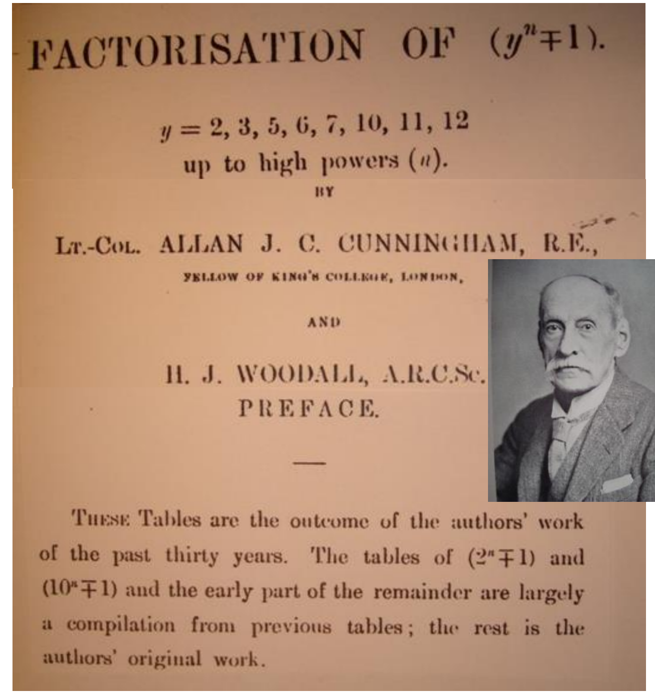

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attacking a single RSA modulus may not be economically viable, but factoring lots of them may become an attractive proposition if effort can be shared

our crypto salespitch

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- if target moduli known in advance: computational inefficiencies (that do not occur for "special" numbers) make regular "one-by-one" approach more efficient

Number field sieve applied to target composite n

1. polynomial selection: degree $d > 1$, integer $m \approx n^{1/d}$, radix m representation of $n = f_d m^d + \ldots + f_1 m + f_0$ leads to $f(X) = \Sigma_i f_i X^i \in \mathbf{Z}[X]$ with $f(m) \equiv 0 \bmod n$

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if many n: amortization leads to overall savings

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1. $d > 1$, $m \approx 2^{L/d}$, collect $a, b \in \mathbf{Z} \times \mathbf{Z}_{\geq 0}$ s.t. $a - mb$ smooth
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unless we reverse the roles of $a-mb$ and $b^d f(a/b)$:
single f may cater to several m-values (and thus n-values)

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(1+7 factored by ECM – 10+3 of 11+6 factored so far)

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most matrix steps **not** "as usual" but with "double product"

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total 7500 core years (86.7% EPFL, 12.8% MSR, 0.5% CH grid), about half of
estimated 14’000 - 21’000 core years’ individual effort

Time saved, in theory, all "heuristic expected"

$$L(c) = \exp((c+o(1))(\log(n))^{1/3}(\log(\log(n)))^{2/3}), \quad n \to \infty$$

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$L(c) = \exp((c+o(1))(\log(n))^{1/3}(\log(\log(n)))^{2/3})$, $n \to \infty$

NFS: factors n in time $L((64/9)^{1/3}) \approx L(1.923)$

SNFS: factors "special" n in time $L((32/9)^{1/3}) \approx L(1.526)$

Time saved, in theory, all "heuristic expected"

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- after $L(2.007)$ preparation, factor n in $L(1.639)$
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"Special" factorization factory:

- after $L(1.615)$ preparation, factor n in $L(1.211)$
- advantageous if $> L(1.615-1.526) = L(0.089)$ distinct "special" (and suitable) n values must be factored

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- more such n values: individual time reduces to $L(0.763)$ while preparation time $\to \infty$ (before amortization)

Conclusion

- showed practical value of Coppersmith's factorization
  factory for known set of "special" numbers
- current implementations of various smoothness tests
  suggest that new ideas are needed for
  a 1024-bit RSA factorization factory
  (for larger RSA moduli it may already be worthwhile)

Conclusion

- showed practical value of Coppersmith's factorization
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- 10011000001101011000000011110010001111011111110000000100101110000111101010010100010100001011010100001110001000100
  10111001011111111011100111000001101100001010111010110111010101000101101001110100101001001001011111101000101110 0110
  111101001111110110000100000010001110010101001000110100010110000000001101000110010011000111000010000000111000010000
  0011001
  is a 346-bit prime that divides the 1193-bit number
  1111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111
  1111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111
  1111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111
  1111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111
  1111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111
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  1111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111
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  1111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111
  1111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111
  1111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111
  111111111111111111111111111111111111111111111111111111111111
  (this is the current special number field sieve record)